Learning Non-linear Ranking Functions for Web Search using Probabilistic Model Building GP

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***Abstract*—Ranking the set of search results according to their relevance to a user query is an important task in an Information Retrieval (IR) systems such as a Web Search Engine. Learning the optimal ranking function for this task is a challenging problem because one must consider complex non-linear interactions between numerous factors such as the novelty, authority, contextual similarity, etc. of thousands of documents that contain the user query. We model this task as a *non-linear* ranking problem, for which we propose *Rank-PMBGP*, an efficient algorithm to learn an optimal non-linear ranking function using Probabilistic Model Building Genetic Programming. We evaluate the proposed method using the LETOR dataset, a standard benchmark dataset for training and evaluating ranking functions for IR. In our experiments, the proposed method obtains a Mean Average Precision (MAP) score of 0.291, thereby significantly outperforming a non-linear baseline approach that uses Genetic Programming.**

I. Introduction

The amount of information available on the Web continues to grow exponentially by the day. It is no longer the case that the information we seek do not exist in the Web, but the problem is to find the relevant information from a large collection of documents. Web search engines provide an efficient interface to the Web. In a typical search session, a user enters one or more keywords to a search engine, which we refer to as a *query*. The search engine then returns a ranked set of results in the descending order of their relevance to the user query. Often, there are millions of documents that match a user query and the task of ranking those documents according to their relevance is a challenging but an important one to a Web search engine [1].

Accurate ranking of search results is an important task for a Web search engine. If a search engine often ranks irrelevant results as the top hits, then users get dissatisfied with that search engine and will soon move to more competitive search engines. Adverts are a main source of income for search engines. If a search engine does not display relevant adverts to user queries, the users will not click on those adverts, resulting in reduced revenue to the search engine. Therefore, the problem of ranking search results in information retrieval systems have received much attention from both academia as well as from the industry. In particular, the Learning to Rank (LETOR) project by Microsoft Research[1], and the Yahoo! learning to rank challenge[2] are noteworthy initiatives.

[1]http://research.microsoft.com/en-us/um/beijing/projects/letor/
[2]http://learningtorankchallenge.yahoo.com

Ranking search results retrieved for a user query is a difficult problem because of several challenges. First, there are numerous factors a search engine must take into consideration when determining the rank of a search result such as the content of the document (i.e. web page) (i.e. whether the document contains the words in the query), the structure of the document (i.e. whether the query appears in the title of the document, its body or in an anchor text pointing to the document), link structure (i.e. the number of in-bound and out-bound links to the document), authority (i.e. encyclopedic resources edited by numerous authors vs. personal blogs), and novelty (i.e. how often does the content in a document is revised and the last updated time). The exact combination of those heterogenous factors that produces the best possible ranking for a set of documents is not obvious. Second, the ranking function must be simple enough to compute and scalable to be used in a Web search engine. If a search engine takes a long time to rank the retrieved set of documents, it might lead to user dissatisfaction. Third, any approach that learns a ranking function for information retrieval must be able to efficiently learn from large datasets. Search engines record the search sessions such as the queries entered by the users and the search results they visit. This process enables us to collect large datasets that can be used as training data to learn ranking functions. For example, LETOR dataset contains over 25 million documents annotated with rank information for numerous queries entered by users in real-world search sessions.

We propose a method to learn a *non-linear* ranking function for information retrieval using Probabilistic Model Building Genetic Programming (PMBGP). We refer to our proposed method as *Rank-PMBGP*. PMBGP is an extension of genetic programming (GP) using probabilistic models. Although there have been several approaches proposed in prior work that can learn a *linear* ranking function, to the best of our knowledge, *Rank-PMBGP* is the first approach to learn non-linear ranking functions for information retrieval using evolutionary algorithms. The ability to learn non-linear ranking functions is particularly important for information retrieval. For example, consider the two features: the number of occurrences of the query in the document, and the authority of the document. If the number of occurrences of a query in a document is high, it indicates that the document is relevant to the query. However, sometimes spam web sites include popular queries to attract web traffic. Therefore, the number of occurrences of a query in a document is a good indicator of relevance only when the authority of the document is high. Such conditional dependencies among factors that influence the rank of a document can be captured only by non-linear ranking functions. Consequently, non-linear ranking function learning methods have shown superior performance over methods that are limited to learning only linear ranking functions [2].

Our contributions in this paper can be summarized as follows.

- We propose a method to learn non-linear ranking functions for information retrieval using probabilistic model building genetic programming. We propose two techniques to overcome overfitting associated with non-linear learning algorithms: the use of validation data for fitness evaluation, and feature selection to reduce the function space.
- We evaluate the proposed method using a standard benchmark dataset that was previously proposed for evaluating learning to rank methods for information retrieval. Our experimental results show that the proposed method significantly outperforms a baseline method that uses genetic programming to learn non-linear ranking functions. Moreover, the performance reported by the proposed method is comparable to that of the state-of-the-art learning to rank methods that use evolutionary algorithms. However, unlike prior work based on evolutionary algorithms, our method can learn non-linear combinations of features.

II. Background

A. Learning to rank

The problem of learning a function that can assign ranks for a set of items arises in numerous contexts. For example, in a web search scenario, we must rank the set of documents (i.e. web pages) according to their relevance to the query entered by a user. As a result of the increasing importance of web search engines as an efficient interface to the vast amounts of information available on the Web, the problem of learning to rank has received special attention in the information retrieval community. There are two main stages involved in learning to rank for information retrieval: (a) learning a ranking function using a labeled dataset (i.e. *training stage*), (b) applying the learnt ranking function to assign ranks to a set of documents retrieved for a user-query (i.e. *ranking stage*).

In the training stage, a ranking function learning algorithm is presented with a ranked list of documents retrieved for a particular query. To formally define the learning problem, let us denote the set of queries by $\mathcal{Q} = \{q_1, q_2, \ldots, q_{|\mathcal{Q}|}\}$, in which we use the notation, $|\mathcal{Q}|$, to represent the number of elements (i.e. cardinality) in the set $\mathcal{Q}$. Likewise, we represent the set of documents by $\mathcal{D} = \{d_1, \ldots, d_{|\mathcal{D}|}\}$. Then, the training dataset can be represented as a set of query-document pairs, $(q_i, d_j) \in \mathcal{Q} \times \mathcal{D}$, in which each query-document pair, (q_i, d_j), is assigned with a relevance judgement, $y(q_i, d_j)$, indicating the relevance of the document d_j to the query q_i. The relevance judgement $y(q_i, d_j)$ can be expressed in several ways. The simplest approach is to indicate a binary relevance $y(q_i, d_j) \in \{0, 1\}$, depending on whether the document d_j is relevant to the query q_i (i.e. $y(q_i, d_j) = 1$), or irrelevant (i.e. $y(q_i, d_j) = 0$). Alternatively, one can assign a real-valued relevance judgement that can be used to induce a total ranking among the documents retrieved for a particular query.

Web search engines record each search session in a log file called the search log to obtain relevance judgements. *Clickthrough* [3] is popular method to easily collect a large collection of relevance judgements. In clickthrough approach, a search engine records the urls that were clicked by a user among all the urls displayed to that user for a particular query. For example, let us assume three documents d_1, d_2, and d_3 are shown to a user, in that order, as the list of search results for a particular query q. Moreover, let us assume that the user did not click on the first document d_1 and instead clicked on the second document d_2. This action is recorded by the search engine before it directs the user to d_2. In the clickthrough approach to obtaining relevance judgements, we assume that the document d_2 is more relevant to the query q than the document d_1. Therefore, a relevance score is assigned such that $y(q, d_2) > y(q, d_1)$. However, no relevance judgements are inferred for the documents such as d_3 that are not clicked on and appears below the lowest ranked document that is clicked by a user for a particular query (i.e. d_2 in this example). Because web search engines are used by millions of users on a daily basis, we have large search logs from which we can extract large training datasets to learn ranking functions.

The goal of learning to rank is to learn a function $f(q, d)$ that assigns a ranking score indicating the degree of relevance of a document d to a query q. First a query-document pair (q, d) is represented by a feature vector $\boldsymbol{\phi}(q, d)$. Numerous features have been proposed in prior work in learning to rank such as the number of occurrences of the query q in the title or the body of a document d, and PageRank [4] of d. In Section IV-A, we detail the numerous features that are used for training by our proposed method. Most prior work on learning to rank model the ranking function $f(q, d)$ as a linearly weighted combination of the features in $\boldsymbol{\phi}(q, d)$ as follows,

$$f(q, d) = \boldsymbol{w}^\top \boldsymbol{\phi}(q, d). \quad (1)$$

Here, $\boldsymbol{w}$ is a vector representing the weight associated with a particular feature in $\boldsymbol{\phi}(q, d)$. We refer to the ranking function given by Equation 1 as a *linear* ranking function because it does not consider non-linear combinations of features in $\boldsymbol{\phi}(q, d)$. In contrast, our proposed method learns a non-linear combinations of features, thus having a greater expressiveness. Specifically, we model the problem of learning to rank as a search problem, where we must find the optimal non-linear combination of features representing a query-document pair that assigns ranking scores similar to the scores assigned in the training data.

Before we explain the search algorithm that we use to find the optimal non-linear combination of features, we must first devise a method to evaluate the *fitness* of a given combination of features. Let us denote the ranking function corresponding to some non-linear combination of features in $\boldsymbol{\phi}(q,d)$ by $f(q,d)$. Then we can use $f(q,d)$ to assign ranking scores to the set of documents $\mathcal{D}(q)$ retrieved for the query q. Next, we can compare the list of ranked documents produced by $f(q,d)$ against the ranks assigned to the documents in $\mathcal{D}(q)$ in the training dataset. The degree to which the two lists of ranks agree is an indicator of the fitness of the combination of features we use to define $f(q,d)$. Next, in Section II-B, we introduce the evaluation measures that are popularly used in the information retrieval community to asses the agreement between a list of ranked documents by a ranking algorithm and that by a human annotator.

B. Evaluation Measures

To evaluate a ranking produced by an algorithm for a set of documents retrieved for a particular query, we can compare it against the ranking induced by the scores assigned by a human annotator for those documents. Precision at position n ($P@n$), Mean Average Precision (MAP), and Normalized Discounted Cumulative Gain (NDCG) are three widely used rank evaluation measures in the information retrieval community. All those evaluation measures are in the range $[0,1]$, where a method that produces the exact ranking as in the gold standard achieves the score of 1. Next, we describe each of those evaluation measures in detail.

Precision at rank n ($P@n$) [5] measure is defined as the proportion of the relevant documents among the top n-ranked documents,

$$P@n = \frac{\text{No. of relevant docs in top } n \text{ results}}{n}. \quad (2)$$

Average precision averages the $P@n$ at over different n values to produce a single measure for a given query as follows,

$$AP = \frac{\sum_{n=1}^{N}(P@n \times rel(n))}{\text{No. of relevant docs for this query}}. \quad (3)$$

Here, N is the number of retrieved documents, and $rel(n)$ is a binary function that returns the value 1 if the n-th ranked document is relevant to the query under consideration and 0 otherwise. Mean average precision (MAP) is computed as the average of AP over all queries in the dataset.

NDCG considers the reciprocal of the logarithm of the rank assigned to relevant documents. For a ranked list of documents retrieved for a query, NDCG value at position n, $NDCG@n$, is computed as follows,

$$NDCG@n = Z_n \sum_{j=1}^{n} \frac{2^{r(j)}-1}{\log(1+j)}. \quad (4)$$

Here, $r(j)$ is the rating of the j-th document in the ranked list, and the normalization constant Z_n is chosen such that a perfectly ranked list would obtain an NDCG@n score of 1. Specifically, it is given by,

$$Z_n = \frac{1}{\sum_{j=1}^{n} \frac{1}{\log(1+j)}}. \quad (5)$$

Algorithm 1 Genetic Programming(GP)

```
1: $g \leftarrow 0$
2: $\mathcal{P}_g \leftarrow$ Initialize $M$ individuals
3: while terminate criterion is $False$ do
4:     Evaluate $\mathcal{P}_g$
5:     $g \leftarrow g+1$
6:     $\mathcal{S}_g \leftarrow$ Select $N$ ($N \leq M$) superior individuals
7:     $\mathcal{P}_g \leftarrow$ Copy $M \times P_e$ elite individuals
8:     $\mathcal{P}_g \leftarrow$ Generate $M(1-P_e)$ individuals from $\mathcal{S}_g$, using
       crossover or mutation
9: end while
10: return $\mathcal{P}_g$
```

We use Mean Average Precision (MAP) as the fitness function because it provides a single value that we can use to determine the fitness of the ranking function $f(q,d)$. All three measures, MAP, NDCG, and P@n, are used to compare the proposed ranking function learning method against previously proposed methods in Section IV-B.

C. Genetic Programming

Genetic Programming (GP) [6] is a widely used and successful method for optimizing non-linear combinations of features represented by tree structures. First, GP randomly generates M tree structures each corresponding to some non-linear combination of features. In subsequent iterations, individual tree structures are evaluated using some fitness function and the top $M \times P_e$ individuals with the highest fitness values are retained to the next generation. Here, P_e denotes the elite rate that determines the number of individuals retained for the next generation. From those retained individuals, GP randomly selects N individuals and performs *mutation* and *cross-over* to produce offspring. Mutation replaces a subtree in a single individual with a different subtree, whereas crossover partitions individuals into constituent subtrees and exchanges subtrees between different individuals. Over the generations, subtree structures that correspond to salient feature combinations are retained in the population, which are referred to as *building blocks*. The above-mentioned procedure is repeated until some pre-defined termination criterion is met. The pseudo code for GP is shown in Algorithm 1. Therein, P_g denotes the population (i.e. set of individuals) at the g-th generation, and S_g is an elite individual selected for reproduction at the g-th generation.

D. Probabilistic Model Building GP

Probabilistic model building GPs (PMBGP) are a variant of Estimation of Distribution Algorithms (EDA) [7], which are generative models based evolutionary computation algorithms for optimizing tree structures. PMBGPs estimate probability distributions using individuals that have the highest fitness values. New individuals are generated by sampling from the estimated probability distribution. PMBGPs can be categorized into two groups. The first type of PMBGPs exploit Probabilistic Context Free Grammar (PCFG) to learn subtree building

TABLE I
TYPES OF NODES USED IN THE PROPOSED RANK-PMBGP.

Node name	Node type	Meaning
S_f	function (trunk)	the set of function nodes {+,-,*}
S_v	terminal (leaf)	the set of variable (feature) nodes
S_c	terminal (leaf)	the set of constant nodes

blocks [8], whereas the second type of PMBGPs use prototype trees, which extend EDAs proposed for one dimensional arrays to handle tree structures [9]. This property of prototype tree-based PMBGPs enables us to incorporate techniques devised in the field of EDAs. For example, sampling of individuals can be done using Loopy Belief Propagation (LBP) [10]. Using the notation we used in Algorithm 1, we show the pseudo code for PMBGP in Algorithm 2. Although PMBGPs have shown better performance than GPs in benchmark problems [11], comparatively to GPs, PMBGPs are yet to be applied to large-scale real-world problems such as the learning to rank for information retrieval, which we study in this paper. We note that besides PMBGP, there are other methods to model the combinations of features for a learning task such as Genetic Network Programming (GNP) [12] and its extension by EDA [13]. Considering more expressive structures such as graphs is in an interesting future research direction.

III. PROPOSED NON-LINEAR RANK LEARNING METHOD: RANK-PMBGP

We propose Rank-PMBGP, a method to learn non-linear ranking functions for information retrieval using PMBGP. Specifically, we use Program Optimization with Linkage Estimation (POLE) [14] as the PMBGP method. POLE is a prototype tree based PMBGP method, which first translates tree structures to one-dimensional arrays and then apply EDAs to those arrays. Individuals in POLE are initialized using GROW, where P_F is the selection rate for functions. POLE uses truncate selection, in which $M \times P_s$ individuals are selected and used for constructing a Bayesian network and estimating parameters. POLE estimates multivariate dependencies between nodes using Bayesian networks. POLE uses Expanded Parse Trees (EPT) [15] to represent the chromosomes, thereby reducing the number of symbols in the tree trunk. EPT pushes terminal nodes on the tree trunk to the leaf nodes using a special function node L. Given a list of arguments as the input, the function L returns the first argument. Therefore, in POLE, the symbols on the trunk are limited to functions. This property of POLE simplifies the task of learning a Bayesian network. This is particularly important in learning to rank for information retrieval because the number of terminal symbols (features) in our task is much higher than that in benchmark problems such as MAX [16], or Royal Tree [17] for which PMBGPs have been applied. The types of nodes used in Rank-PMBGP are summarized in Table I. Rank-PMBGP considers non-linear combinations of features by using multiplication (shown by * in Table I) as a function node. We use MAP as fitness function in Rank-PMBGP.

Algorithm 2 Probabilistic Model Building GP (PMBGP)

```
1: $g \leftarrow 0$
2: $\mathcal{P}_g \leftarrow$ Initialize $M$ individuals
3: while terminate criterion is $False$ do
4:     Evaluate $\mathcal{P}_g$
5:     $g \leftarrow g + 1$
6:     $\mathcal{S}_g \leftarrow$ Select $N$ $(N \leq M)$ superior individuals
7:     $\mathcal{D}_g \leftarrow$ Estimate distribution from $\mathcal{S}_g$
8:     $\mathcal{P}_g \leftarrow$ Copy $M \times P_e$ elite individuals
9:     $\mathcal{P}_g \leftarrow$ Sampling $M(1 - P_e)$ individuals from $D_g$
10: end while
11: return $\mathcal{P}_g$
```

Rank-PMBGP consists of five steps as shown below.

Step 1: Input
Receive train, validation, and test data as the input. Initialize the parameters in PMBGP.

Step 2: Training
For a feature combination p, we compute $\mathrm{MAP}(p, \mathrm{Train})$, the MAP value we obtain if we rank the documents in the train data using the ranking function corresponding to p. We use $\mathrm{MAP}(p, \mathrm{Train})$ as the fitness function in Algorithm 2 (Lines 3, 6, 8, 10).

Step 3: Output
After the PMBGP routine in Algorithm 2 has terminated, we select the individual p^* that maximizes the following,

$$\mathrm{MAP}(p, \mathrm{Train}) + \mathrm{MAP}(p, \mathrm{Validation})$$

among all the individuals p in the final population. Here, $\mathrm{MAP}(p, \mathrm{Validation})$ is the MAP of p over the validation dataset. As described later in Section IV-A, we use a set of queries with documents ranked according to their relevancy to those queries, as a validation dataset separately to the train dataset. The use of validation data helps us to overcome overfitting to train data.

Step 4: Ranking
Rank the documents in the test dataset using the non-linear ranking function corresponding to p^*.

IV. EXPERIMENTS AND RESULTS

A. Dataset

We use the LETOR (version 2.0) benchmark dataset [18] that has been widely used in prior work on learning to rank for information retrieval. The LETOR version 2.0 consists of TD2003 and TD2004 datasets, which were part of the topic distillation task of the Text REtrieval Conference (TREC) in year 2003 and 2004. TD2003 dataset contains 50 queries and TD2004 dataset contains 75 queries. The document collection contains $1,053,110$ documents together with $11,164,829$ hyperlinks and is based on a January, 2002 crawl of the .gov domain. Topic distillation aims to find a list of documents relevant to a particular topic. The TREC committee provides judgements for the topic distillation task. For each query in TD2003 and TD2004 datasets, there are about $1,000$

TABLE II
FEATURES IN THE LETOR TD2003 AND TD2004 DATASETS.

Category	Feature	No. of features
Content (low-level)	tf [5]	4
	idf [5]	4
	dl [5]	4
	tfidf [5]	4
Content (high-level)	BM25 [19]	4
	LMIR [20]	9
Hyperlink	PageRank [4]	1
	Topical PageRank [23]	1
	HITS [21]	2
	Topical HITS [23]	2
	HostRank [22]	1
Hybrid	Hyperlink-base relevance propagation [24]	6
	Sitemap-based relevance propagation [25]	2
Total		44

TABLE III
SETTINGS FOR THE PROPOSED RANK-PMBGP METHOD

Parameters/Nodes	Settings
P_s	if population size is larger than 5000 use 0.05 otherwise use 0.2
P_e	if population size is larger than 5000 use 1 otherwise use 0.005
P_F	0.9
S_f	{+,-,*} (all function takes two arguments)
S_v	11 features (id : name) 5: dl of URL 7: HITS hub 8: HostRank 9: idf of body 10: idf of anchor 11: idf of title 12: idf of URL 18: LMIRJM of anchor 21: LMIRDIR of extracted title 23: LMIRABS of title 39: Hyperlink base score propagation (weighted in-link) }
S_c	$\{0.2, 0.4, 0.6, 0.8, 1.0\}$
The number of terminal symbols	16
depth limitation	8

documents listed. Each query-document pair is given a binary judgement indicating whether a document is relevant or non-relevant for a particular query.

A query-document pair in the LETOR dataset is represented using 44 features as shown in Table II. The features include numerous ranking heuristics popularly used in the information retrieval community. The set of features includes low-level features such as, term frequency (tf), inverse document frequency (idf), document length (dl) combinations of low-level features such as tf*idf [5], as well as high-level features such as BM25 [19] and LMIR [20]. Hyperlink structure provides useful clues about the relevancy of a web page. Consequently, several features are computed using the hyperlink information in LETOR datasets such as PageRank [4], HITS [21], HostRank [22], topical PageRank and topical HITS [23]. Following the standard practice, all features are normalized to $[0, 1]$ range prior to training. For the TD2003 and TD2004 datasets, we define two values of ratings 0 and 1 respectively corresponding to relevant and non-relevant documents in order to compute NDCG scores. In our evaluations, we report the average values taken over all the queries in a dataset as $P@n$ and $NDCG@n$.

B. Results

One major challenge involved in learning the optimal non-linear combination of a large number of features (e.g. 44 in TD2003) using a comparatively smaller number of training instances (e.g. 50 queries) is overfitting. Because the possible hypothesis space is much more complex with non-linear functions relatively to linear functions, extra care must be taken to reduce overfitting. As already mentioned under the Step 3 in Section III, the use of validation data to select the best individual provides a partial solution to the overfitting problem. As a complementary solution, we reduce the complexity of the search space by performing feature selection prior to learning a non-linear ranking function with those features. In addition to reducing overfitting, feature selection speed ups the training process, enabling us to use large training datasets.

To select salient features for ranking, first, we train a linear ranking function using all the features in a dataset. We use the linear ranking function learning algorithm proposed by Bollegala et al. [26] for this purpose. This method learns a weight for each feature, which represents the influence that feature has upon the ranking produced by the linear combination of features as expressed in Equation 1. We select the features with weights larger than 2 for Rank-PMBGP. Table III shows the set of selected features and the values of all parameters in Rank-PMBGP. To avoid any biases due to initialization, we report the average results over 10 repetitions for all the experiments described in this paper.

We compare the performance of the proposed method against several previously proposed methods and a baseline.

RankSVM [27] extends Support Vector Machines, originally proposed for binary classification, to rank learning.

RankBoost [28] combines a variety of ranking scores based on AdaBoost. Individual features in LETOR are considered as weak classifiers for boosting. Performance for both RankSVM and RankBoost are obtained from the official LETOR report [18].

SwarmRank [29] learns a linear ranking function by maximizing MAP using Particle Swarm Optimization (PSO).

RankGP [30] applies GP to the rank learning problem. However, this method is limited to learning linear ranking functions.

RankDE [26] is a Differential Evolution (DE) based ranking algorithm. It finds the optimal linear combination of features that maximizes MAP over a dataset. To our knowledge, RankDE is the current state-of-the-art among the methods that use evolutionary algorithms.

Baseline Method: Following RankGP, we implement a method using GP that can learn non-linear ranking functions. This baseline serves two purposes. First, it demonstrates the difference in performance between GP and PMBGP in the context of learning to rank. Second, it acts

TABLE IV
PARAMETERS IN THE **BASELINE METHOD**.

Parameter	Definition	Value
P_e	Elitist Reproduction Rate	Only 1 individual
P_c	Crossover Rate	Initial value = 0.95, then change dynamically using AMRT
P_m	Mutation Rate	Initial value = 0.05, then change dynamically using AMRT
size_t	Tournament Size	5
P_F	Functional Selection Rate	0.9

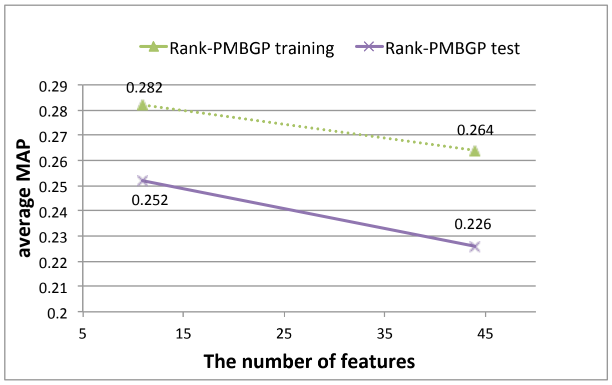


Fig. 1. The effect of feature selection on Rank-PMBGP.

as a non-linear version of Rank-GP, thereby demonstrating any gain in performance due to non-linearities. We use tournament selection with Adaptive Mutation Rate Tuning (AMRT) [31] for this baseline. AMRT increases the mutation rate and decreases the crossover rate when the population is likely to converge. The parameter values for this baseline are shown in Table IV.

Rank-PMBGP is our proposed method.

Table V compares the performance of the above-mentioned methods on TD2003 using MAP, P@n, and NDCG. On MAP, we see that **Rank-PMBGP** outperforms **RankGP** as well as the **Baseline** method. This shows the superiority of non-linear ranking functions over linear ranking functions in the context of learning to rank for information retrieval. Moreover, only **RankDE** performs better than **Rank-PMBGP** on TD2003. Although **RankDE** can only learn linear ranking functions, its good performance can be attributable to the DE-based learning algorithm. Indeed, an interesting future research direction would be to explore the possibility of using DE to learn non-linear ranking functions. Overall, our proposed **Rank-PMBGP** significantly outperforms **RankSVM**, **RankBoost**, **SwamRank**, and the **Baseline** methods at 0.05 significance level according to on a paired t-test.

Figure 1 shows the effect of feature selection on our proposed Rank-PMBGP. We see a steep drop in MAP when the number of features is increased, indicating a severe overfitting. This shows that feature selection is an important step when learning non-linear combinations of features.

We study the effect of the number of fitness evaluations has upon the overall performance of the proposed **Rank-PMBGP** and the **Baseline**. From Figure 2, we see that on both TD2003 as well as TD2004, **Rank-PMBGP** performs better than the **Baseline** at the end of the training. In particular, on TD2003, initially the performance of **Rank-PMBGP** is lower than that of the **Baseline**. However, the performance of the **Baseline** method does not improve over the iterations whereas **Rank-PMBGP** does. The fact that **Rank-PMBGP** constantly outperforms the **Baseline** on two different datasets shows the reliability of the proposed method.

V. RELATED WORK

Learning to rank methods can be divided into three approaches:pointwise, pairwise, and listwise. The pointwise approach [32], [33] deals with each query-document pair independently during entire training and ranking. Because the pointwise approach dismisses the relative preferences between query-document pairs for the same query, it often results in poor performances.

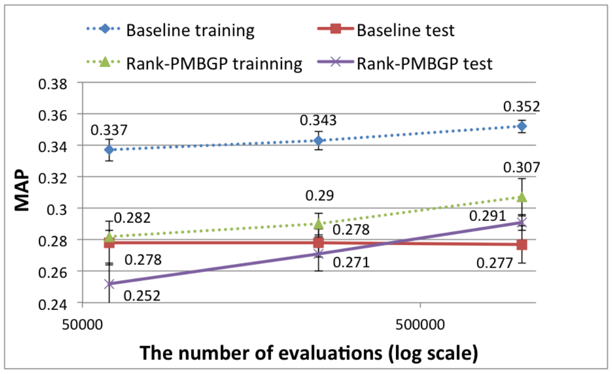
Pairwise approach [27], [28], [34], [35] considers partially ordered preferences between a pair of documents for the same query. Representative methods for pairwise rank learning are Ranking Support Vector Machines (RankSVM) [27], RankBoost [28], and RankNet [34]. Despite the wide popularity of pairwise approaches, they considers only a pair of documents at a time, thus ignoring the remainder of the documents retrieved for a query.

Listwise approach [36]–[39] considers the entire set of documents retrieved for a particular query during training, thereby overcoming the above-mentioned disfluencies in the pointwise and pairwise approaches. In information retrieval, we must apply the learnt ranking function to induce a total ordering for a set of documents retrieved for a query. Listwise approach models this situation well. Therefore, we follow the listwise approach in this paper to learn a ranking function from a given set of training data. Different loss functions have been used in prior work on listwise rank learning, leading to numerous algorithms such as ListNet [37] (cross entropy), RankCosine [36] (cosine loss), and ListMLE [39] (likelihood loss). However, these methods do not directly optimize the evaluation criteria used in information retrieval such as MAP or NDCG, and instead approximate them via the above-mentioned loss functions. In contrast, **Rank-PMBGP** directly optimizes those evaluation criteria, without requiring any approximations.

Fan et al. [40]–[42] proposed a GP-based approach to learn a term-weighting formula by combining numerous features. First, they use an expression tree data structure to represent a term-weighting formula, and then apply GP to select the best performing function. Numerous operators such as addition, subtraction, multiplication, division, square root, logarithm etc. are considered. Almeida et al. [43] propose *Combined Component Approach* (CCA), a GP-based ranking function, that combines several term-weighting components such as term frequency, collection frequency, etc. to generate ranking functions. Yeh et al. [44] propose RankGP, a learning to rank method using GP. RankGP regards linear ranking functions as

TABLE V
COMPARISON OF THE PERFORMANCES REPORTED BY DIFFERENT LEARNING TO RANK METHODS ON TD2003.

Method	RankSVM	RankBoost	SwarmRank	RankGP	RankDE	Baseline	Rank-PMBGP
MAP	0.256	0.212	0.209	0.283	0.339	0.277	0.291
P@1	0.420	0.260	0.453	0.520	0.600	0.528	0.548
P@2	0.350	0.270	0.330	0.420	0.400	0.444	0.468
P@3	0.340	0.240	0.269	0.370	0.333	0.380	0.405
P@4	0.300	0.230	0.223	0.330	0.300	0.336	0.352
P@5	0.264	0.220	0.207	0.280	0.280	0.294	0.294
P@6	0.243	0.210	0.188	0.270	0.250	0.265	0.267
P@7	0.234	0.211	0.185	0.250	0.243	0.250	0.247
P@8	0.233	0.193	0.173	0.240	0.237	0.229	0.232
P@9	0.218	0.182	0.164	0.230	0.222	0.214	0.215
P@10	0.206	0.178	0.151	0.220	0.210	0.199	0.204
NDCG@1	0.420	0.260	0.453	0.520	0.600	0.528	0.548
NDCG@2	0.370	0.280	0.343	0.450	0.445	0.463	0.486
NDCG@3	0.379	0.270	0.307	0.420	0.388	0.413	0.438
NDCG@4	0.363	0.272	0.284	0.390	0.356	0.378	0.396
NDCG@5	0.347	0.279	0.278	0.380	0.336	0.345	0.353
NDCG@6	0.341	0.280	0.271	0.370	0.310	0.321	0.329
NDCG@7	0.340	0.287	0.273	0.360	0.300	0.306	0.311
NDCG@8	0.345	0.282	0.270	0.350	0.292	0.288	0.295
NDCG@9	0.342	0.282	0.267	0.350	0.279	0.275	0.280
NDCG@10	0.341	0.285	0.263	0.350	0.267	0.261	0.269



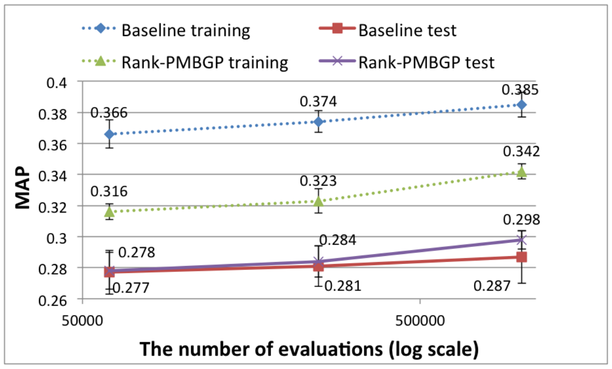


Fig. 2. The effect of the number of evaluations on the performance. TD2003 dataset on the left and TD2004 dataset on the right.

individuals and use adaptive mutation training method [31]. Diaz-Aviles et al. [29] propose SwamRank, a ranking method that uses particle swam optimization (PSO). They use the LETOR benchmark dataset and learn a linear combination of different features that represent a query-document pair to maximize MAP on train data.

Although most existing methods for learning to rank learns only linear ranking functions, there are couple of notable efforts to learn non-linear ranking functions. [41] creates a non-linear ranking function using GP, and shows that it outperforms BM25 using only simple features such as term frequency(tf), inverse document frequency(idf), and document length(dl). They do not evaluate their method on the LETOR dataset that has richer set of features. Friedman et al. [45], [46] proposed a method to learn a non-linear rank function using Gradient Boosted Decision Trees (GBDT). GBDT was shown to be a strong baseline at Yahoo! Learning to Rank Challenge [2]. However, GBDT has not been evaluated on the LETOR benchmark dataset.

VI. CONCLUSION

We proposed Rank-PMBGP, a method to learn non-linear ranking functions for information retrieval using the Probabilistic Model Building GP algorithm POLE. Rank-PMBGP directly optimizes MAP, without requiring any convex approximations. We proposed two methods to overcome the overfitting problem that is common to methods that learn non-linear combinations of features. First, we select the best individual based on the MAP scores on train dataset as well as on an independent validation dataset. Second, we perform feature selection using differential evolution, and then use the selected features in Rank-PMBGP. We evaluated the proposed method using LETOR benchmark dataset. Our experimental results show that our proposed Rank-PMBGP method outperforms a GP-based baseline as well as numerous previously proposed learning to rank methods. Moreover, we showed that the proposed feature selection method can accurately select a subset of features, thereby avoiding overfitting to train data.

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